Theoretical View on $K \to \pi\nu\bar{\nu}$

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The Old View

- Ten years ago, when discussing neutral and charged $K \to \pi\nu\bar{\nu}$, it was viewed as a clean constraint on the CKM matrix.
- The tacit underlying assumption was that only SM particles contribute to the loops.
- It was convenient to combine all CKM factors in Wolfenstein ways.

The New View

- Over the next ten years, we expect other, SM tree-level, processes to determine CKM with ~1% precision.
- Over the next ten years, we expect to observe new particles at the LHC.
- We want new and different insights on their dynamics from the loops of $K \to \pi\nu\bar{\nu}$.

$K_L \to \pi^0 \nu\bar{\nu}$

- Schematically, the (SM) branching ratio is

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) \propto r_{K_L} \mathrm{BR}(K^+ \to \pi^0 e^+ \nu) \times \frac{\alpha^2}{\sin^4 \theta_W} \times \left[\frac{\mathrm{Im}\, V_{ts}^* V_{td}}{|V_{us}|}\right]^2 \times [X(m_t, \alpha_s)]^2$$

  where r_{K_L} describes isospin breaking.

- The largest uncertainty comes from the CKM factor.

- Using CKM unitarity and dropping $|V_{tb}| - 1$:

$$\frac{\mathrm{Im}\, V_{ts}^* V_{td}}{|V_{us}|} = -\frac{|V_{cb}|\,\mathrm{Im}\, V_{ub}}{|V_{us}|} = \frac{|V_{cb}||V_{ub}|\sin\delta_{KM}}{|V_{us}|} = \frac{(A\lambda^2)(A\lambda^3\eta)}{\lambda}$$

  so we want to forecast the uncertainty of all 4 basic CKM parameters over the next several years.

- Let's look at direct determinations; global analysis could shrink errors further.

Lattice QCD

- Much of the prospect for improving $|V_{xy}|$ comes from lattice QCD, especially in concert with semileptonic decays of *K* and *B* mesons.
- Two-day December workshop with USQCD, BaBar, CLEO, CDF, DØ, and BobT: http://www.usqcd.org/lattice-experiment2007.html.
- Estimates informed by talks there.

$|V_{us}|$: Andreas Jüttner

- The current (PDG) uncertainty is around 1%, from K_{l3} and K_{l2} decays:
  - K_{l3}: need form factor $f_+(0)$
  - K_{l2}: need decay constant (ratio) f_K (f_K/f_π)
- Both on track to reduce uncertainties to 0.5% "any day now."
- Don't use λ^8, where $\lambda = |V_{us}|$ (for errors).

$|V_{cb}|$: Jack Laiho

- The current (PDG, HFAG) uncertainty is 1.7%, from inclusive $B \to X_c l\nu$.
- Unquenched lattice QCD calculation for exclusive $B \to D^* l\nu$ has 2.4% error.
- Imperfect agreement must be resolved:

$$|V_{cb}|^{\text{ex}} = (38.7 \pm 0.7 \pm 0.9) \times 10^{-3}$$
$$|V_{cb}|^{\text{in}} = (41.7 \pm 0.4 \pm 0.6) \times 10^{-3}$$

next loop for inclusive complete soon.

$|V_{ub}|$: Ruth Van de Water

- Inclusive methods may stop at 5% (2%).
- The current error budget for exclusive $B \to \pi l \nu$ has several contributions of 1–7%.
- Lattice QCD probably needs two phases, one to get the (quadrature sum) total down to 4-5%; the next to 1-2%.
- Challenging, but feasible; Super *B* factory.

$\sin \delta_{KM}$: LHC*b*

- LHC*b* forecasts an error on $\gamma = \delta_{KM}$ of
  - 5° in 2.5 fb^{-1}
  - 2.5° in 10 fb^{-1}
- This corresponds to a 1% (0.6%) error in $\sin \delta_{KM}$, since $\delta_{KM} \approx 80°$.
- See http://lhcb-doc.web.cern.ch/lhcb-doc/presentations/conferencetalks/postscript/2007presentations/MCalviFlavourPhysics.pdf

Total

$$2\sqrt{3 \times (0.5)^2 + 2^2} = 4\%$$

rate → 2

$|V_{cb}|$, $|V_{us}|$, $\sin \delta_{KM}$ → $3 \times (0.5)^2$

$|V_{ub}|$ → 2^2

Even if 2% for $|V_{ub}|$ is optimistic, I think this uncertainty will come with a Super B factory, and I don't see why the kaon experiments should wait for that.

$K^+ \to \pi^+ \nu\bar{\nu}$

- Schematically, the (SM) branching ratio is

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) \propto r_{K^+} \mathrm{BR}(K^+ \to \pi^0 e^+ \nu) \times \frac{\alpha^2}{\sin^4 \theta_W} \times \sum_l \left| \frac{V_{ts}^* V_{td}}{V_{us}} X(m_t, \alpha_s) + \frac{V_{cs}^* V_{cd}}{V_{us}} X_{\mathrm{NL}}(m_c, m_l, \alpha_s) \right|^2$$

where r_{K^+} describes isospin breaking.

- Same X as before; X_{NL} sums logs.

m_c

- Flavianet reckons that $m_c = 1.30(5)$ GeV (i.e., 4%) leads to 5% uncertainty in BR$^+$.
- Inclusive $B \to X_c l\nu$ can get m_c to 5% [Bigi].
- Unquenched lattice QCD calculations with nonperturbative (or else 3 loop pert.) renormalization could cut this in half.
- So 3% theoretical uncertainty in BR$^+$ is hard to forecast, but easily so in BR$^+$/BR$_L$.

BSM

- New "beyond the SM" particles change the short-distance dynamics:
  - CKM × X += (new FV) × X_{new};
  - if (new FV) $\propto$ CKM, that's called MFV
- Solve BR_L for $X(m_t)$, generalize to $X(m_c)$, plug into BR+, and see if it agrees with experiment: favor or kill MFV.

- By the time you have 1000-event, the LHC experiments will (we all hope) have seen new particles.
- Models to explain them will be developed.
- Every model will have its own $X(v)$, where the $v = \{m_t,\ \alpha_s,$ new couplings & masses$\}$.
- Every model can be favored or killed by BR_L and BR^+.

Summary

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- They measure the short-distance functions, denoted $X(v)$.
- So we can call this series of measurements Project X.