

# 1 Kaon decays

## 1.1 Introduction

The rare decays  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$  play an important role in the search for the underlying mechanism of flavour mixing and CP violation [1–3]. As such they are excellent probes of physics beyond the Standard Model (SM). Among the many rare  $K$ - and  $B$ -decays, the  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$  modes are unique since their SM branching ratios can be computed to an exceptionally high degree of precision, not matched by any other flavour-changing neutral current (FCNC) process involving quarks.

The main reason for the exceptional theoretical cleanness of the  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$  decays is the fact that, within the SM, these processes are mediated by electroweak amplitudes of  $\mathcal{O}(G_F^2)$ , described by  $Z^0$ -penguins and box diagrams which exhibit a power-like GIM mechanism. This property implies a severe suppression of non-perturbative effects, which is generally not the case for meson decays receiving contributions of  $\mathcal{O}(G_F\alpha_s)$  (gluon penguins) and/or  $\mathcal{O}(G_F\alpha_{em})$  (photon penguins), which therefore have only a logarithmic-type GIM mechanism. A related important virtue, following from this peculiar electroweak structure, is the fact that  $K \rightarrow \pi\nu\bar{\nu}$  amplitudes can be described in terms of a single effective operator, namely

$$Q_{sd}^{\nu\bar{\nu}} = (\bar{s}_L\gamma^\mu d_L)(\bar{\nu}_L\gamma_\mu\nu_L) . \quad (1)$$

The hadronic matrix elements of  $Q_{sd}^{\nu\bar{\nu}}$  relevant for  $K \rightarrow \pi\nu\bar{\nu}$  amplitudes can be extracted directly from the well-measured  $K \rightarrow \pi e\nu$  decay rates, taking into account tiny Isospin Breaking (IB) corrections [4]. The latter have recently been estimated beyond the the leading order and turn out to be a negligible source of uncertainty [5].

In the case of  $K_L \rightarrow \pi^0\nu\bar{\nu}$ , which is CP-violating and dominated by the dimension-six top quark contribution, the SM Short-Distance (SD) dynamics is then encoded in a perturbatively calculable real function  $X$  that multiplies the CKM factor  $\lambda_t = V_{ts}^*V_{td}$ . In the case of  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  also a charm quark contribution proportional to  $\lambda_c = V_{cs}^*V_{cd}$  has to be taken into account, but the recent NNLO QCD calculation of the dimension-six charm quark corrections [6, 7] and the progress in the evaluation of dimension-eight charm and Long-Distance (LD) up quark effects [8] elevated the theoretical cleanness of  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  almost to the level of  $K_L \rightarrow \pi^0\nu\bar{\nu}$ . More details will be given in the following.

The important virtue of  $K \rightarrow \pi\nu\bar{\nu}$  decays is that their clean theoretical character remains valid in essentially all extensions of the SM and that  $Q_{sd}^{\nu\bar{\nu}}$ , due to the special properties of the neutrinos, remains the only relevant operator. Consequently, in most SM extensions the New Physics (NP) contributions to  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$  can be parametrized in a model-independent manner by just two parameters, the magnitude and the phase of the function [9]

$$X = |X|e^{i\theta_X} , \quad (2)$$

that multiplies  $\lambda_t$  in the relevant effective Hamiltonian. In the SM,  $|X| = X_{\text{SM}}$  and  $\theta_X = 0$ .

The parameters  $|X|$  and  $\theta_X$  can be extracted from  $\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$  without hadronic uncertainties, while the function  $X$  can be calculated in any extension of

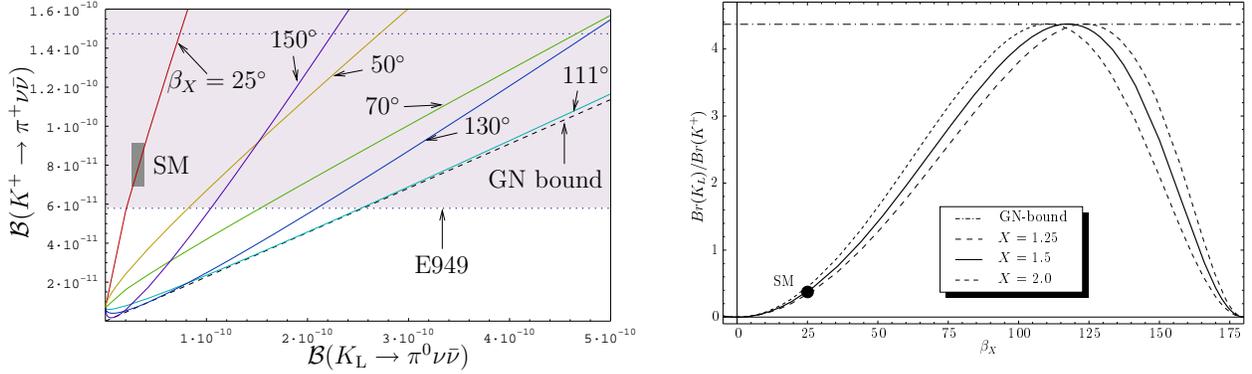


Figure 1: Left:  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  vs.  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  for various values of  $\beta_X = \beta - \theta_X$  (including E949 data) [13]. The dotted horizontal lines indicate the lower part of the experimental range [10–12] and the grey area the SM prediction. We also show the Grossman-Nir (GN) bound [14]. Right: The ratio of the  $K \rightarrow \pi \nu \bar{\nu}$  branching ratios as a function of  $\beta_X$  for  $|X| = 1.25, 1.5, 2.0$ . The horizontal line is again the GN bound.

the SM within perturbation theory. Of particular interest is the ratio

$$\frac{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} = \left| \frac{X}{X_{\text{SM}}} \right|^2 \left[ \frac{\sin(\beta - \theta_X)}{\sin \beta} \right]^2. \quad (3)$$

Bearing in mind that  $\beta \approx 21.4^\circ$  shows that  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is a very sensitive function of the new phase  $\theta_X$ . The pattern of the two  $K \rightarrow \pi \nu \bar{\nu}$  branching ratios as a function of  $\theta_X$  is illustrated in Fig. 1 (left). We note that the ratio of the two modes shown in Fig. 1 (right) depends very mildly on  $|X|$  and therefore provides an excellent tool to extract the non-standard CP-violating phase  $\theta_X$ .

## 1.2 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the SM

After summation over the three lepton families the SM branching ratios for the  $K \rightarrow \pi \nu \bar{\nu}$  decays can be written as

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = \kappa_+ \left[ \left( \frac{\text{Im} \lambda_t}{\lambda^5} X_{\text{SM}} \right)^2 + \left( \frac{\text{Re} \lambda_t}{\lambda^5} X_{\text{SM}} + \frac{\text{Re} \lambda_c}{\lambda} (P_c + \delta P_{c,u}) \right)^2 \right], \quad (4)$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = \kappa_L \left( \frac{\text{Im} \lambda_t}{\lambda^5} X_{\text{SM}} \right)^2, \quad (5)$$

where  $\lambda = |V_{us}|$ , while  $\kappa_+ = (5.165 \pm 0.025) \cdot 10^{-11} (\lambda/0.225)^8$  and  $\kappa_L = (2.231 \pm 0.013) \cdot 10^{-10} (\lambda/0.225)^8$  include the IB corrections in relating  $K \rightarrow \pi \nu \bar{\nu}$  to the  $K \rightarrow \pi e \nu$  rates [5].<sup>1</sup>

<sup>1</sup> This value of  $\kappa_+$  corresponds to the photon-inclusive  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  rate. The (tiny) modifications due to possible photon-energy cuts can be found in Ref. [5]

The dimension-six top quark contribution  $X_{\text{SM}} = 1.464 \pm 0.041$  [6, 7] accounts for around 63% and almost 100% of the total rates. It is known through NLO [16, 17], with a scale uncertainty of slightly below 1%. In  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , dimension-six charm quark corrections and subleading dimension-eight charm and LD up quark effects, characterized by  $P_c = 0.38 \pm 0.04$  [6, 7] and  $\delta P_{c,u} = 0.04 \pm 0.02$  [8], amount to moderate 33% and a mere 4%. Light quark contributions are negligible in the case of the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decay [18].

Taking into account all the indirect constraints from the latest global Unitarity Triangle (UT) fit, the SM predictions of the two  $K \rightarrow \pi \nu \bar{\nu}$  rates read [19]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.22 \pm 0.84) \cdot 10^{-11}, \quad \mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = (2.76 \pm 0.40) \cdot 10^{-11}. \quad (6)$$

The quoted central value of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  corresponds to  $m_c = 1.3 \text{ GeV}$  and the given error breaks down as follows: residual scale uncertainties (13%),  $m_c$  (22%), CKM,  $\alpha_s$ , and  $m_t$  (37%), and matrix-elements from  $K \rightarrow \pi e \nu$  and light quark contributions (28%). The main source of uncertainty in  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is parametric (83%), while the impact of scales (12%) and IB (5%) is subdominant. SM predictions of  $K \rightarrow \pi \nu \bar{\nu}$  with total uncertainties at the level of 5% or below are thus possible through a better knowledge of  $m_c$ , of the IB in the  $K \rightarrow \pi$  form factors, and/or by a lattice study [20] of higher-dimensional and LD contributions.

While the determination of  $|V_{td}|$ ,  $\sin 2\beta$ , and  $\gamma$  from the  $K \rightarrow \pi \nu \bar{\nu}$  system is without doubt still of interest, with the slow progress in measuring the relevant branching ratios and much faster progress in the extraction of the angle  $\gamma$  from the  $B_s \rightarrow DK$  system to be expected at the LHC, the role of the  $K \rightarrow \pi \nu \bar{\nu}$  system will shift towards the search for NP rather than the determination of the CKM parameters.

In fact, determining the UT from tree-level dominated  $K$ - and  $B$ -decays and thus independently of NP will allow to find the true values of the CKM parameters. Inserting these, hopefully accurate, values in Eqs. (4) and (5) will allow to obtain very precise SM predictions for the rates of both rare  $K$ -decays. A comparison with future data on  $K \rightarrow \pi \nu \bar{\nu}$  may then give a clear signal of potential NP contributions in a theoretically clean environment. Even deviations by 20% from the SM expectations could be considered as signals of NP, while such a conclusion cannot be drawn in most other decays in which the theoretical errors are at least 10%.

## 1.3 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ beyond the SM

### 1.3.1 Minimal Flavour Violation

In models with Minimal Flavour Violation (MFV) [21, 22] both decays are, like in the SM, governed by a single real function  $X$  that can take a different value than in the SM due to new particle exchange in the relevant  $Z^0$ -penguin and box diagrams. Restricting first our discussion to the so-called constrained MFV (CMFV) (see [23]), in which strong correlations between  $K$ - and  $B$ -decays exist, one finds that the branching ratios for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  cannot be much larger than their SM values given in Eq. (6). The 95% probability bounds read [24]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{CMFV}} \leq 11.9 \cdot 10^{-11}, \quad \mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{CMFV}} \leq 4.6 \cdot 10^{-11}. \quad (7)$$

Explicit calculations in a model with one Universal Extra Dimension (UED) [25] and in the littlest Higgs model without  $T$ -parity [27] give explicit examples of this scenario with the branching ratios within 20% of the SM expectations. The latest detailed analysis of  $K \rightarrow \pi\nu\bar{\nu}$  in the Minimal Supersymmetric SM (MSSM) with MFV can be found in [26].

Probably the most interesting property of this class of models is a theoretically clean determination of the angle  $\beta$  of the standard UT, which utilizes both branching ratios and is independent of the value of  $X$  [28, 29]. Consequently, this determination is universal within the class of MFV models and any departure of the resulting value of  $\beta$  from the corresponding one measured in  $B$ -decays would signal non-MFV interactions.

### 1.3.2 Littlest Higgs Model with $T$ -parity

The structure of  $K \rightarrow \pi\nu\bar{\nu}$  decays in the Littlest Higgs model with  $T$ -parity (LHT) differs notably from the one found in MFV models due to the presence of mirror quarks and leptons that interact with the light fermions through the exchange of heavy charged ( $W_H^\pm$ ) and neutral ( $Z_H^0, A_H^0$ ) gauge bosons. The mixing matrix  $V_{Hd}$  that governs these interactions can differ from  $V_{CKM}$ , which implies the presence of non-MFV interactions. Instead of a single real function  $X$  that is universal within the  $K$ -,  $B_d$ - and  $B_s$ -systems in MFV models, one now has three functions

$$X_K = |X_K|e^{i\theta_K}, \quad X_d = |X_d|e^{i\theta_d}, \quad X_s = |X_s|e^{i\theta_s}, \quad (8)$$

that due to the presence of mirror fermions can have different phases and magnitudes. This possibility can have a major impact on the  $K \rightarrow \pi\nu\bar{\nu}$  system, since the correlations between  $K$ - and  $B$ -decays are partly lost and the presence of a large phase  $\theta_K$  can change the pattern of these decays from the one observed in MFV. A detailed analysis [15] shows that both branching ratios can depart significantly from their SM values, and can be as high as  $5.0 \cdot 10^{-10}$ . As shown in Fig. 2 (left), there are two branches of allowed values with strong correlations between both branching ratios within a given branch. In the lower branch only  $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$  can differ substantially from the SM expectations reaching values well above the present central experimental value. In the second branch  $\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$  can be as high as  $5.0 \cdot 10^{-10}$  and  $2.3 \cdot 10^{-10}$ , respectively. Moreover,  $\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})$  can be larger than  $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$  which is excluded within MFV models. Other features distinguishing this model from MFV are thoroughly discussed in [15].

### 1.3.3 Supersymmetry

Within the MSSM with  $R$ -parity conservation, sizable non-standard contributions to  $K \rightarrow \pi\nu\bar{\nu}$  decays can be generated if the soft-breaking terms have a non-MFV structure. The leading amplitudes giving rise to large effects are induced by: i) chargino/up-squark loops [9, 30–32] ii) charged Higgs/top quark loops [33]. In the first case, large effects are generated if the left-right mixing ( $A$  term) of the up-squarks has a non-MFV structure [22]. In the second case, deviations from the SM are induced by non-MFV terms in the right-right down sector, provided the ratio of the two Higgs vacuum expectation values ( $\tan\beta = v_u/v_d$ ) is large ( $\tan\beta \sim 30 - 50$ ).

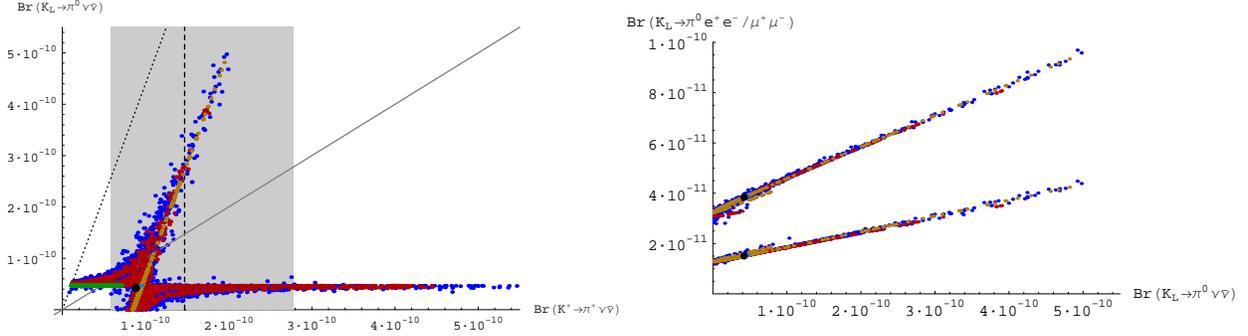


Figure 2: Left:  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  vs.  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  in the LHT model [15]. The shaded area represents the experimental  $1\sigma$ -range for  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ . The GN bound is displayed by the dotted line, while the solid line separates the two areas where  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  is larger or smaller than  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ . Right:  $\mathcal{B}(K_L \rightarrow \pi^0 e^+ e^-)$  (upper curve) and  $\mathcal{B}(K_L \rightarrow \pi^0 \mu^+ \mu^-)$  (lower curve) as functions of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  in the LHT model [15].

The effective Hamiltonian encoding SD contributions in the general MSSM has the following structure:

$$\mathcal{H}_{\text{eff}}^{(\text{SD})} \propto \sum_{l=e,\mu,\tau} V_{ts}^* V_{td} [X_L (\bar{s}_L \gamma^\mu d_L) (\bar{\nu}_{lL} \gamma_\mu \nu_{lL}) + X_R (\bar{s}_R \gamma^\mu d_R) (\bar{\nu}_{lL} \gamma_\mu \nu_{lL})] , \quad (9)$$

where the SM case is recovered for  $X_R = 0$  and  $X_L = X_{\text{SM}}$ . In general, both  $X_R$  and  $X_L$  are non-vanishing, and the misalignment between quark and squark flavour structures implies that they are both complex quantities. Since the  $K \rightarrow \pi$  matrix elements of  $(\bar{s}_L \gamma^\mu d_L)$  and  $(\bar{s}_R \gamma^\mu d_R)$  are equal, the combination  $X_L + X_R$  allows us to describe all the SD contributions to  $K \rightarrow \pi \nu \bar{\nu}$  decays. More precisely, we can simply use the SM expressions for the branching ratios in Eqs. (4) to (5) with the following replacement

$$X_{\text{SM}} \rightarrow X_{\text{SM}} + X_L^{\text{SUSY}} + X_R^{\text{SUSY}} . \quad (10)$$

In the limit of almost degenerate superpartners, the leading chargino/up-squarks contribution is [31]:

$$X_L^{X^\pm} \approx \frac{1}{96} \left[ \frac{(\delta_{LR}^u)_{23} (\delta_{RL}^u)_{31}}{\lambda_t} \right] = \frac{1}{96 \lambda_t} \frac{(\tilde{M}_u^2)_{2L3R} (\tilde{M}_u^2)_{3R1L}}{(\tilde{M}_u^2)_{LL} (\tilde{M}_u^2)_{RR}} . \quad (11)$$

As pointed out in [31], a remarkable feature of the above result is that no extra  $\mathcal{O}(M_W/M_{\text{SUSY}})$  suppression and no explicit CKM suppression is present (as it happens in the chargino/up-squarks contributions to other processes). Furthermore, the  $(\delta_{LR}^u)$ -type mass insertions are not strongly constrained by other  $B$ - and  $K$ -observables. This implies that large departures from the SM expectations in  $K \rightarrow \pi \nu \bar{\nu}$  decays are allowed, as confirmed by the complete analyses in [26, 34]. As illustrated in Fig. 3 (left),  $K \rightarrow \pi \nu \bar{\nu}$  are the best observables to determine/constrain from experimental data the size of the off-diagonal  $(\delta_{LR}^u)$  mass insertions

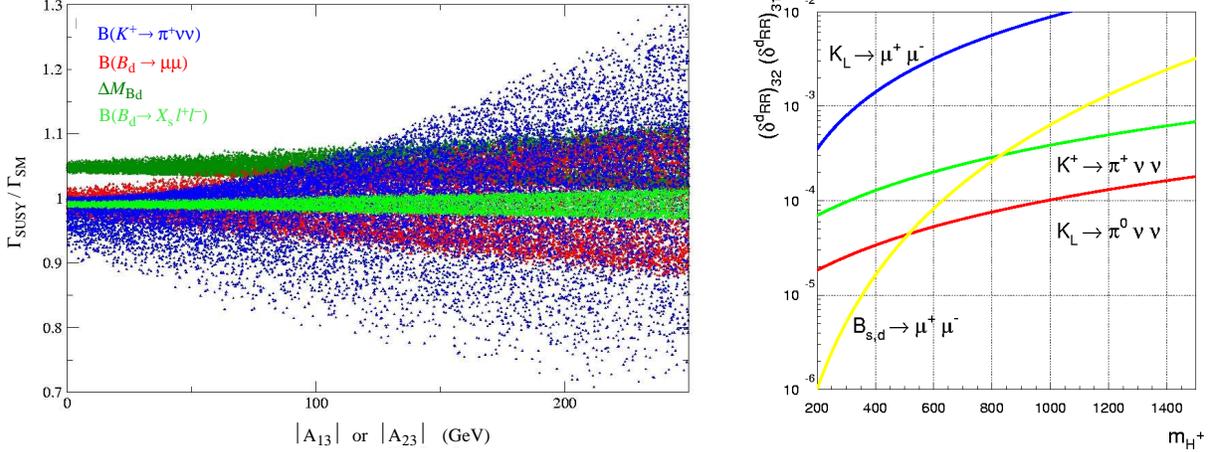


Figure 3: Supersymmetric contributions to  $K \rightarrow \pi \nu \bar{\nu}$ . Left: Dependence of various FCNC observables (normalized to their SM value) on the up-type trilinear terms  $A_{13}$  and  $A_{23}$ , for  $A_{ij} \leq \lambda A_0$  and  $\tan \beta = 2-4$  (other key parameters in GeV:  $\mu = 500 \pm 10$ ,  $M_2 = 300 \pm 10$ ,  $M_{\tilde{u}_R} = 600 \pm 20$ ,  $M_{\tilde{q}_L} = 800 \pm 20$ ,  $A_0 = 1000$ ) [26]. Right: Sensitivity to  $(\delta^d_{RR})_{23} (\delta^d_{RR})_{31}$  of various rare  $K$ - and  $B$ -decays as a function of  $M_{H^+}$ , setting  $\tan \beta = 50$ ,  $\mu < 0$  and assuming almost degenerate superpartners (the bounds from the two  $K \rightarrow \pi \nu \bar{\nu}$  modes are obtained assuming a 10% measurements of their branching ratios while the  $B_{s,d} \rightarrow \mu^+ \mu^-$  bounds refer to the present experimental limits [33]).

or, equivalently, the up-type trilinear terms  $A_{i3}$  [ $(\tilde{M}_u^2)_{iL3R} \approx m_t A_{i3}$ ]. Their measurement is therefore extremely interesting also in the LHC era.

In the large  $\tan \beta$  limit, the charged Higgs/top-quark exchange leads to [33]:

$$X_R^{H^\pm} \approx \left[ \left( \frac{m_s m_d t_\beta^2}{2M_W^2} \right) + \frac{(\delta^d_{RR})_{31} (\delta^d_{RR})_{32}}{\lambda_t} \left( \frac{m_b^2 t_\beta^2}{2M_W^2} \right) \frac{\epsilon_{RR}^2 t_\beta^2}{(1 + \epsilon_i t_\beta)^4} \right] f_H(y_{tH}). \quad (12)$$

where  $y_{tH} = m_t^2/M_H^2$ ,  $f_H(x) = x/4(1-x) + x \log x/4(x-1)^2$  and  $\epsilon_{i,RR} t_\beta = \mathcal{O}(1)$  for  $t_\beta = \tan \beta \sim 50$ . The first term of Eq. (12) arises from MFV effects and its potential  $\tan \beta$  enhancement is more than compensated by the smallness of  $m_{d,s}$ . The second term on the r.h.s. of Eq. (12), which would appear only at the three-loop level in a standard loop expansion can be largely enhanced by the  $\tan^4 \beta$  factor and does not contain any suppression due to light quark masses. Similarly to the double mass-insertion mechanism of Eq. (11), also in this case the potentially leading effect is the one generated when two off-diagonal squark mixing terms replace the two CKM factors  $V_{ts}$  and  $V_{td}$ .

The coupling of the  $(\bar{s}_R \gamma^\mu d_R)(\bar{\nu}_L \gamma_\mu \nu_L)$  effective FCNC operator, generated by charged-Higgs/top-quark loops is phenomenologically relevant only at large  $\tan \beta$  and with non-MFV right-right soft-breaking terms: a specific but well-motivated scenario within grand-unified theories (see e.g. [35, 36]). These non-standard effects do not vanish in the limit of heavy squarks and gauginos, and have a slow decoupling with respect to the charged-Higgs boson

mass. As shown in [33] the  $B$ -physics constraints still allow a large room of non-standard effects in  $K \rightarrow \pi\nu\bar{\nu}$  even for flavour-mixing terms of CKM size (see Fig. 3 right).

## 1.4 Conclusions

The rare  $K \rightarrow \pi\nu\bar{\nu}$  decays are excellent probes of New Physics. Firstly, their exceptional cleanness allows to access very high energy scales. As stressed recently in [15, 26, 37, 38], NP could be seen in rare  $K$  decays without significant signals in  $B_{d,s}$ -decays and, in specific scenarios, even without new particles within the LHC reach. Secondly, if LHC finds NP, its energy scale will be fixed. Then, the measurements of the two  $K \rightarrow \pi\nu\bar{\nu}$  rates would be very helpful in discriminating among NP models.

It is worth stressing that if a deviation from the SM is seen in one of the two  $K \rightarrow \pi\nu\bar{\nu}$  channels, a key independent information about the nature of NP can be obtained also from the two  $K_L \rightarrow \pi^0\ell^+\ell^-$  ( $\ell = e, \mu$ ) modes. The latter are not as clean as the neutrino modes, but are still dominated by SD dynamics and very sensitive to NP [39–42]. Different correlations among these four channels are expected in different NP models (see e.g. Fig. 2). These correlations can be used as powerful tests to shed light on the nature of NP. In all cases where visible effects are found, the information extracted from the four modes is essential to establish the NP flavour structure in the  $s \rightarrow d$  sector. Rare  $K$  decays are thus an integral part, along with  $B$ -physics and collider observables, of the grand project of reconstructing the NP model from data.

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